Effect of Higher Order Chromaticity and Impedance on linear Chromaticity

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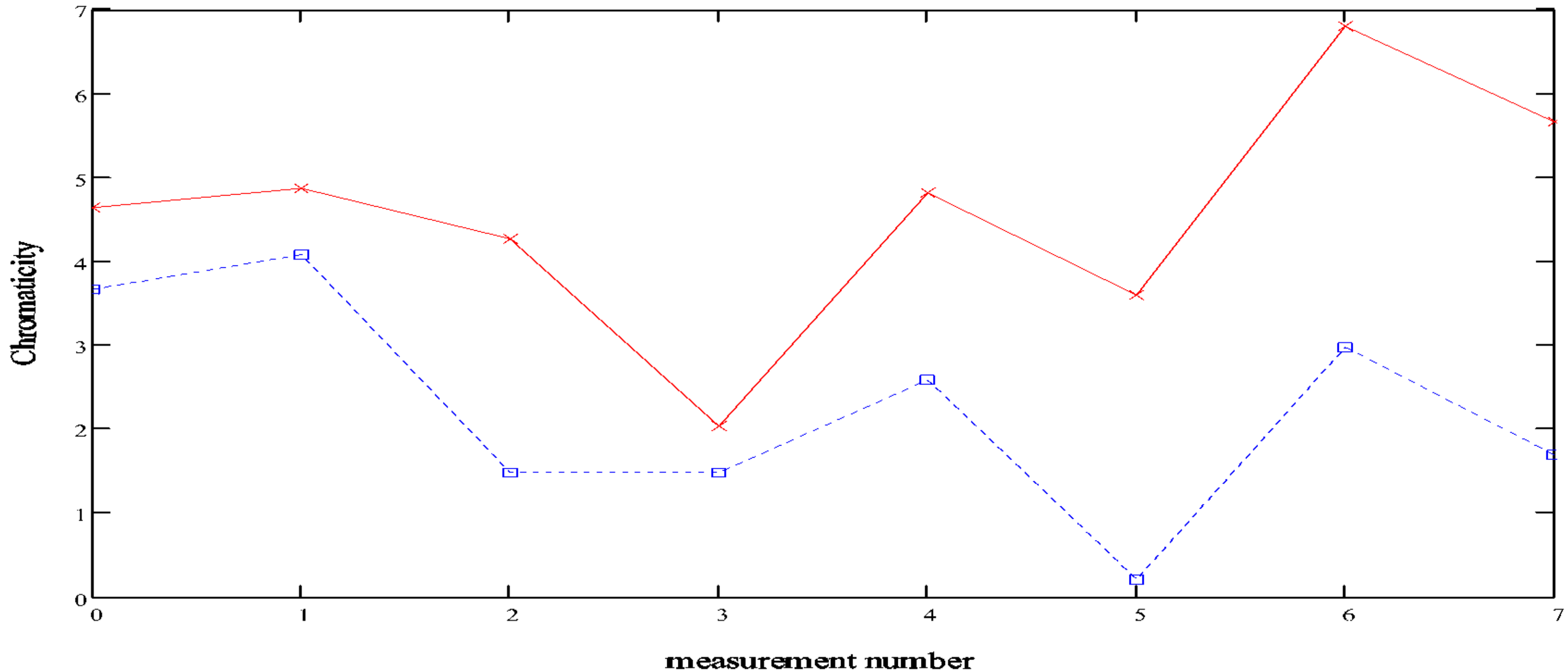
- Motivation : Experience in the Tevatron
- Formalism Developed to help explain the results:
- Results for Tevatron
- Preliminary results for RHIC

Relevant parameters: sigma delta p/p, Coherent tune shift, un-perturbed Chromaticity (1^{st} and 2^{nd} Order), Phase slip factor

Motivation:



In developing several chromaticity applications for the Tevatron C.Y Tan and I had noticed a consistent difference between linear chromaticity measurements for coalesced and uncoalesced proton with precisely the same optics. For several years we didn’t have a good answer as to the cause of this effect. We had explored possible emittance effects and high Dispersion effects, but none of these provided a consistent story. However in simulation work for another paper I had noticed that 2^{nd} order chromaticity together with wakefields could effect the beam in a complex manner.

Dispersion Integral Formalism

$$(-U + jV)^{-1} = \int \frac{\rho(\delta)}{\Omega_c - \omega_n(\delta)} d\delta$$

where we have defined

$$V + jU = \frac{q\beta I Z_\perp}{4\pi R\gamma m Q\omega_0} \qquad \omega_n(\delta) = (n+Q)\omega_0 + [\xi - (n+Q)\eta]\omega_0\delta$$

The solution to the Dispersion Integral with momentum offsets is:

$$(v + ju)^{-1} = e^{-Z^2}\mathrm{erfc}(-jZ)$$

$$u = \frac{\sqrt{\pi}U}{\sigma_\omega}$$

$$v = \frac{\sqrt{\pi}V}{\sigma_\omega}$$

$$Z = \frac{\Omega_c - (n+Q)\omega_0}{\sigma_\omega} - \frac{\delta_0}{\sqrt{2}\sigma_\delta}$$

$$\sigma_\omega = \sqrt{2}[\xi - (n+Q)\eta]\omega_0\sigma_\delta$$

If we introduce different momentum offsets in our distribution the effect is to just move along the existing u-v curve according to the chromaticity.

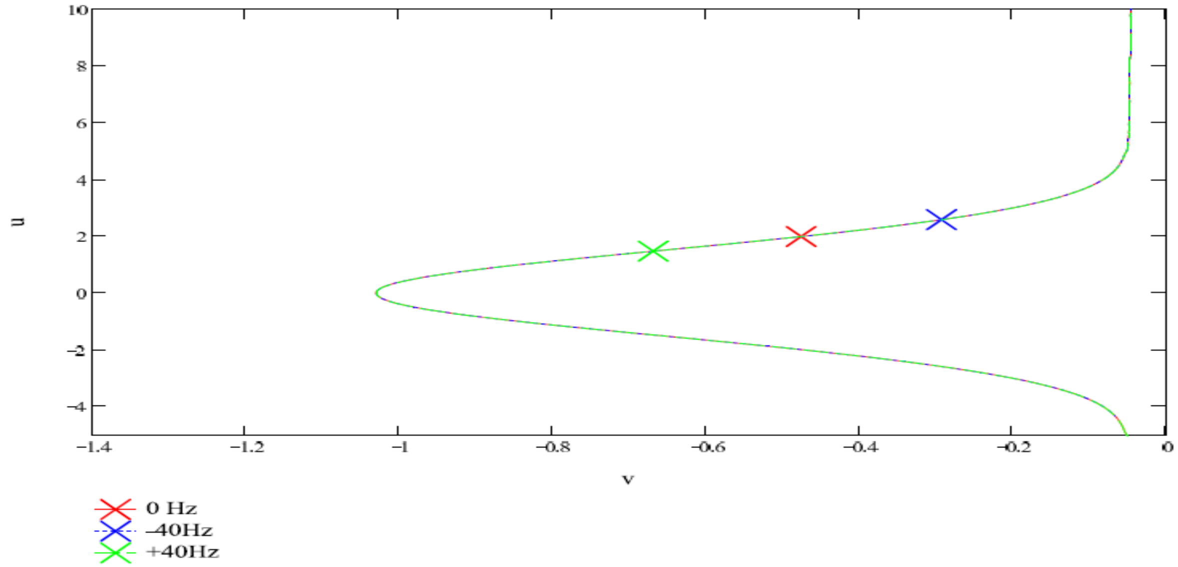


FIG. 1: The normalized u versus v curves for a Gaussian distribution is plotted with the growth rate $\mathrm{Im}(\Omega_c) = -0.031\sigma_\omega$ for three δ_0 offsets (40, 0, -40) Hz . All three curves lie on top of each other. The symbols '$\times$' mark the position of the coherent tune shift for each δ_0.

If we now consider the effects of second order chromaticity, we can expand Eq. 11 to second order in δ to get,

$$\omega_{n,\omega}(\delta) = (n+Q)\omega_0 + \Big[\xi - (n+Q)\eta\Big]\omega_0\delta + \left(\frac{\xi'}{2} - \xi\eta\right)\omega_0\delta^2 \tag{17}$$

The Solution now becomes:

$$\begin{aligned}(v+ju)^{-1} \;&= -\tfrac{j}{2d\pi g}e^{-(g+b)^2}\left(-\pi\mathrm{erfi}(g+b) - e^{4gb}(\pi\mathrm{erfi}(g-b) - \ln(g-b) - \ln(-g+b))\right.\\ &\qquad\qquad \left. - \ln(-g+b) - \ln(g+b)\right)\end{aligned} \tag{21}$$

Were we have defined the following:

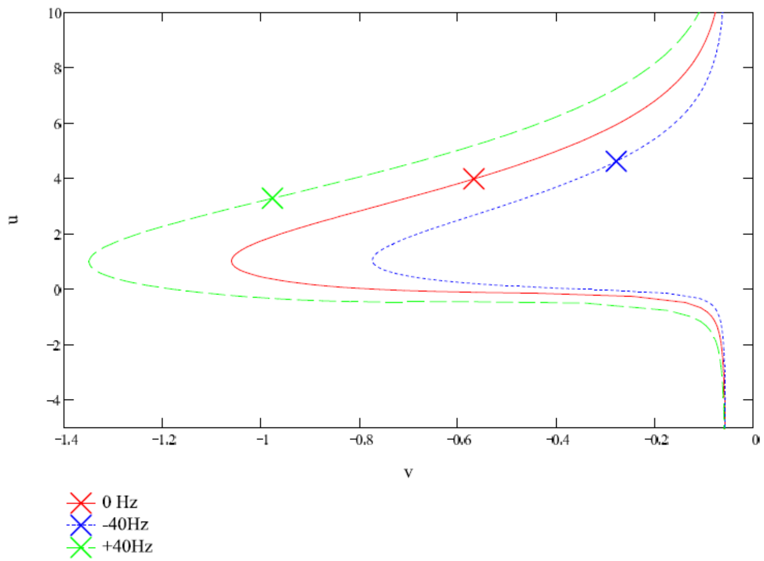
$$d = \frac{2\sigma_\delta^2 a_\omega}{\sigma_\omega}$$

$$a_w = \left(\tfrac{1}{2}\xi' - \xi\eta\right)\omega_0$$

$$b = \frac{1 + \frac{a_\omega 2\sqrt{2}\sigma_\delta \delta_0}{\sigma_\omega}}{2d}$$

$$c = \frac{-Z + \frac{\delta_0^2 a_\omega}{\sigma_\omega}}{d}$$

$$g = \sqrt{b^2 - c}$$



In this case the different momentum generate different u-v curves . This introduces an additional momentum dependence which alters the linear Chromaticity measured.

FIG. 3: The normalized u versus v curves for a Gaussian distribution is plotted with growth rate $\mathrm{Im}(\Omega_c) = -0.031\sigma_\omega$ for three δ_0 offsets (40, 0, -40) Hz in the presence of second order chromaticity set to -4000 units. Unlike Fig. 1, the curves are now separated for each δ_0. The '$\times$' symbols mark the position of the coherent tune shifts for each δ_0.

6D Simulations using BBSIMC

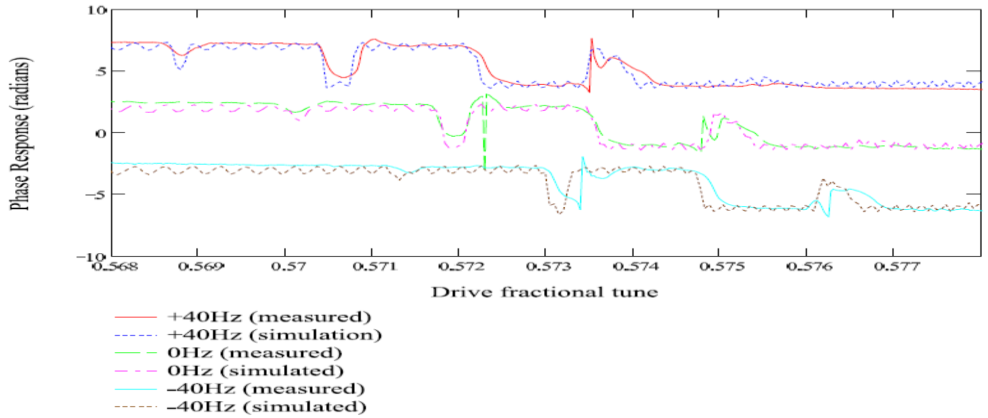


FIG. 8: The phase of the BTF for uncoalesced protons at different frequency offsets (i.e. different δ_0). Overlaid are the simulated results using BBSIMc with 4.7 units of linear chromaticity, -2×10^3 units of second order chromaticity

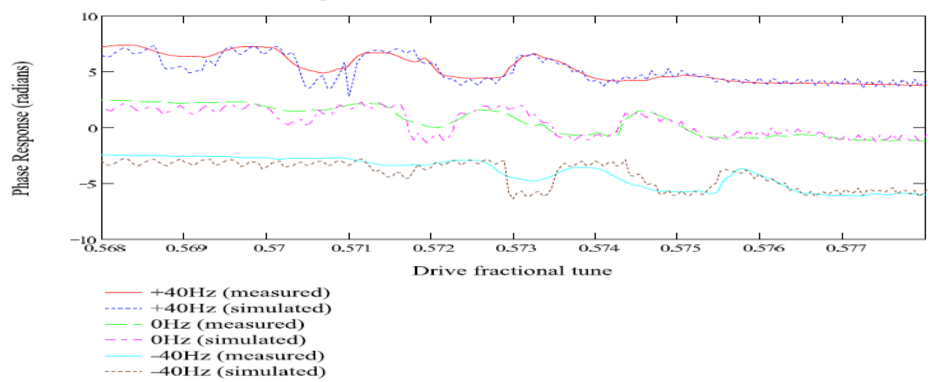


FIG. 9: The phase of the BTF for coalesced protons at different frequency offsets (i.e. different δ_0). Overlaid are the simulated results using BBSIM with 4.7 units of linear chromaticity, -2×10^3 units of second order chromaticity, resistive wall wakefield and the intensity of 3.0×10^{11} protons.

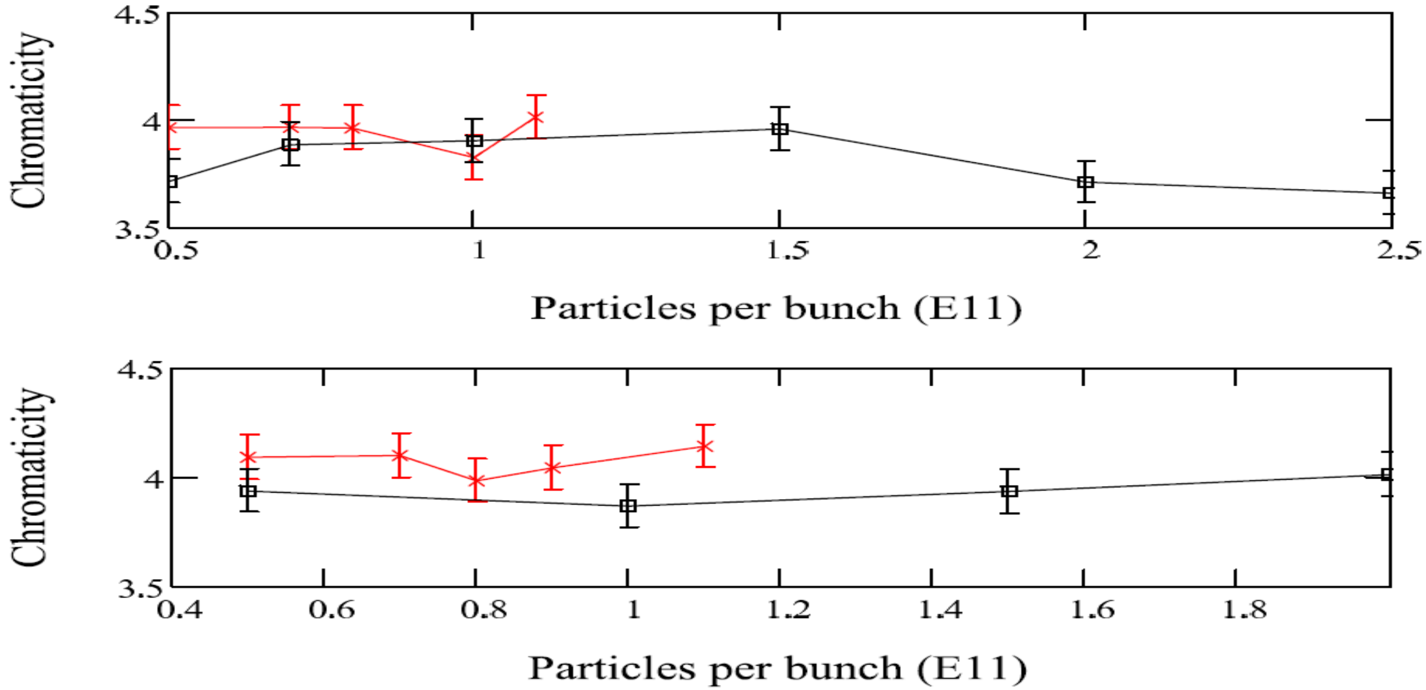
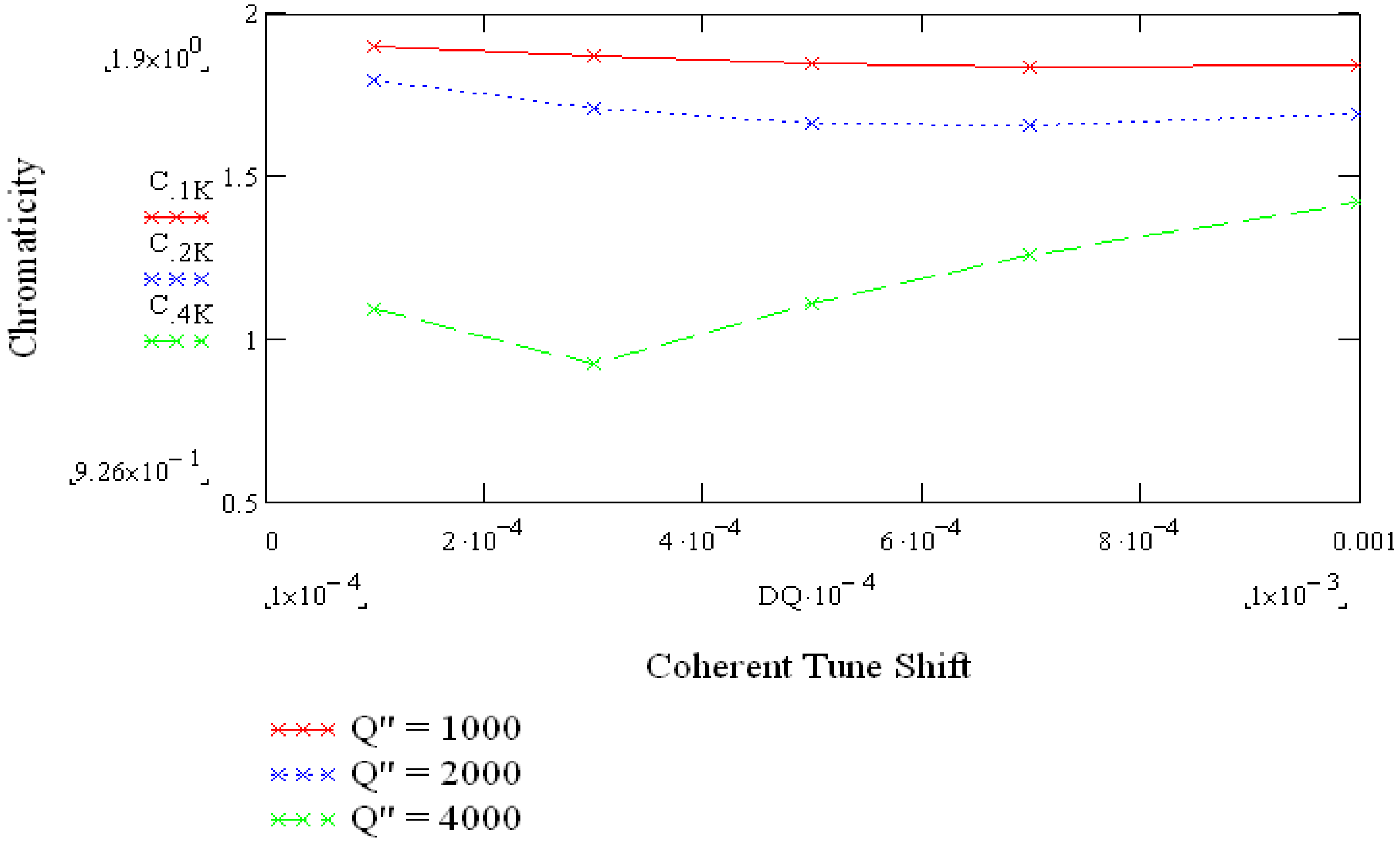


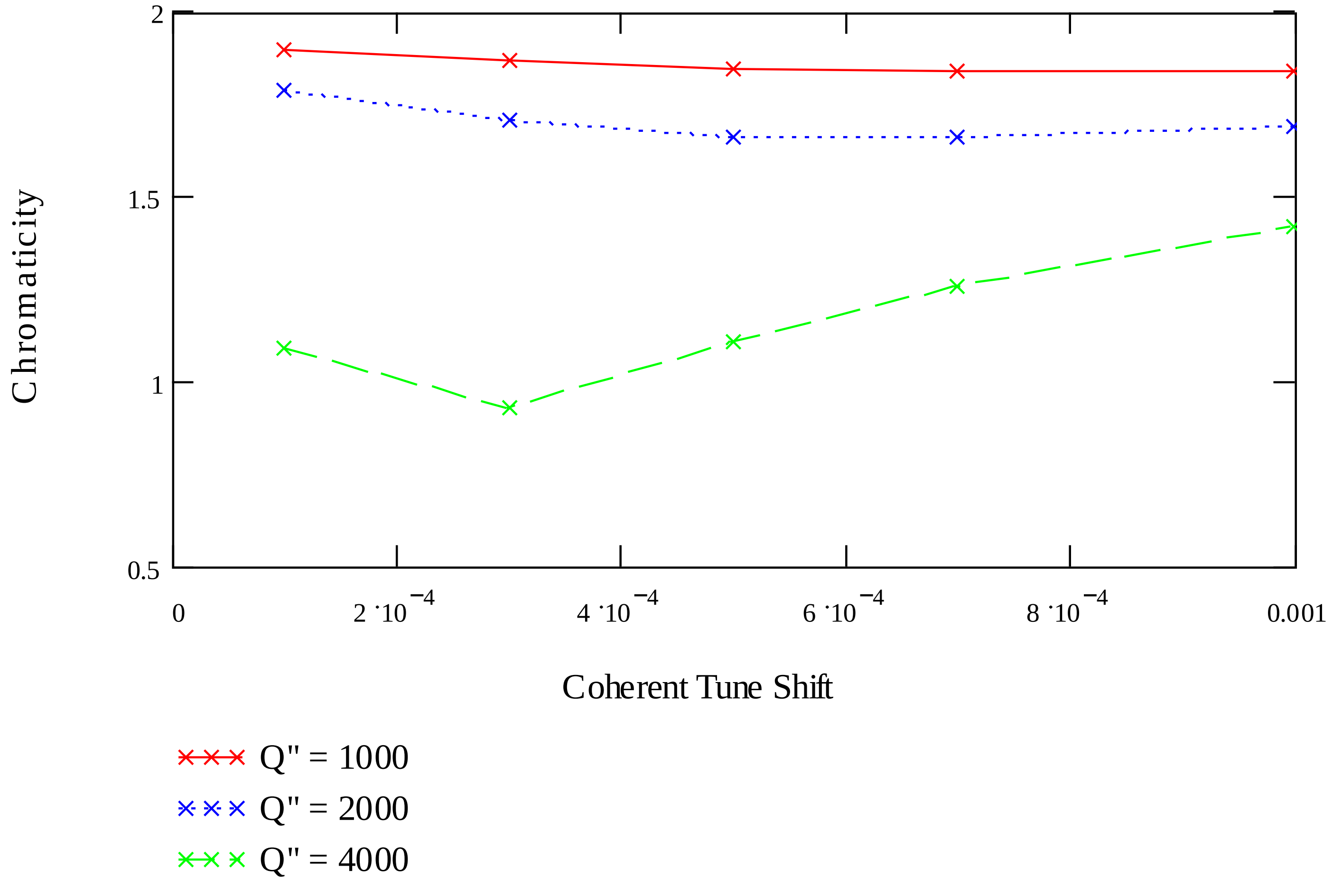
FIG. 10: Plot of the results of the 6D simulations with 3×10^5 particles using BBSIM. The simulation was setup to imitate a chromaticity measurement using BTF method. We used 4.0 units of linear chromaticity. The top plot shows simulations with -4×10^3 units of second order chromaticity, the bottom plot with $+4 \times 10^3$ units of second order chromaticity. In these plots the red trace ("×") show uncoalesced protons, the black trace (□) show coalesced protons with. These are all plotted with intensities up to instability threshold with only transverse resistive wall impedance effects (no longitudinal).

For coalesced protons sigma dp/p = 5e-4 for uncoalesced = 2.5e-4

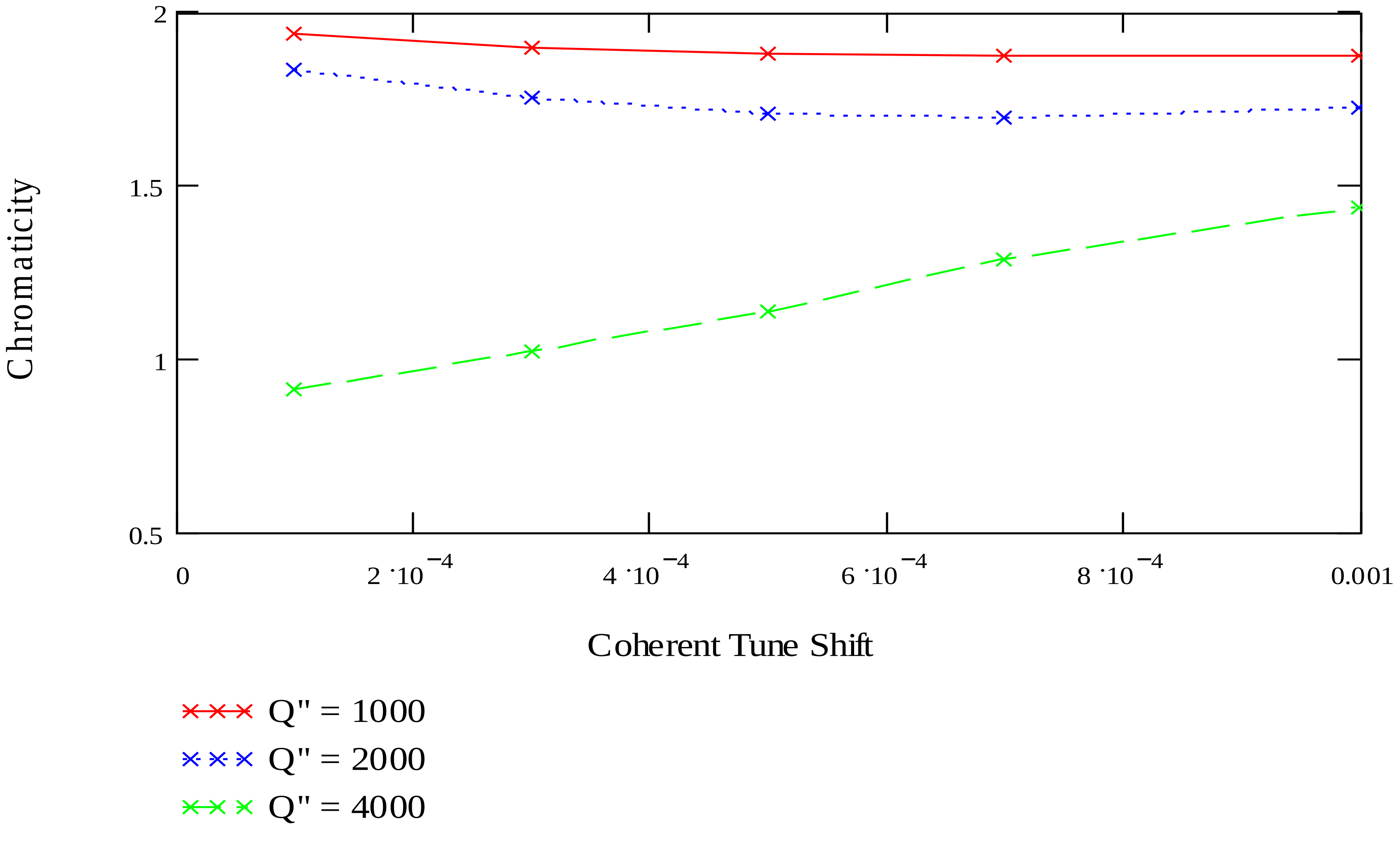
Response of Linear Chromaticty in RHIC (sigma deltap/p = 0.3e-3 Gamma=250)



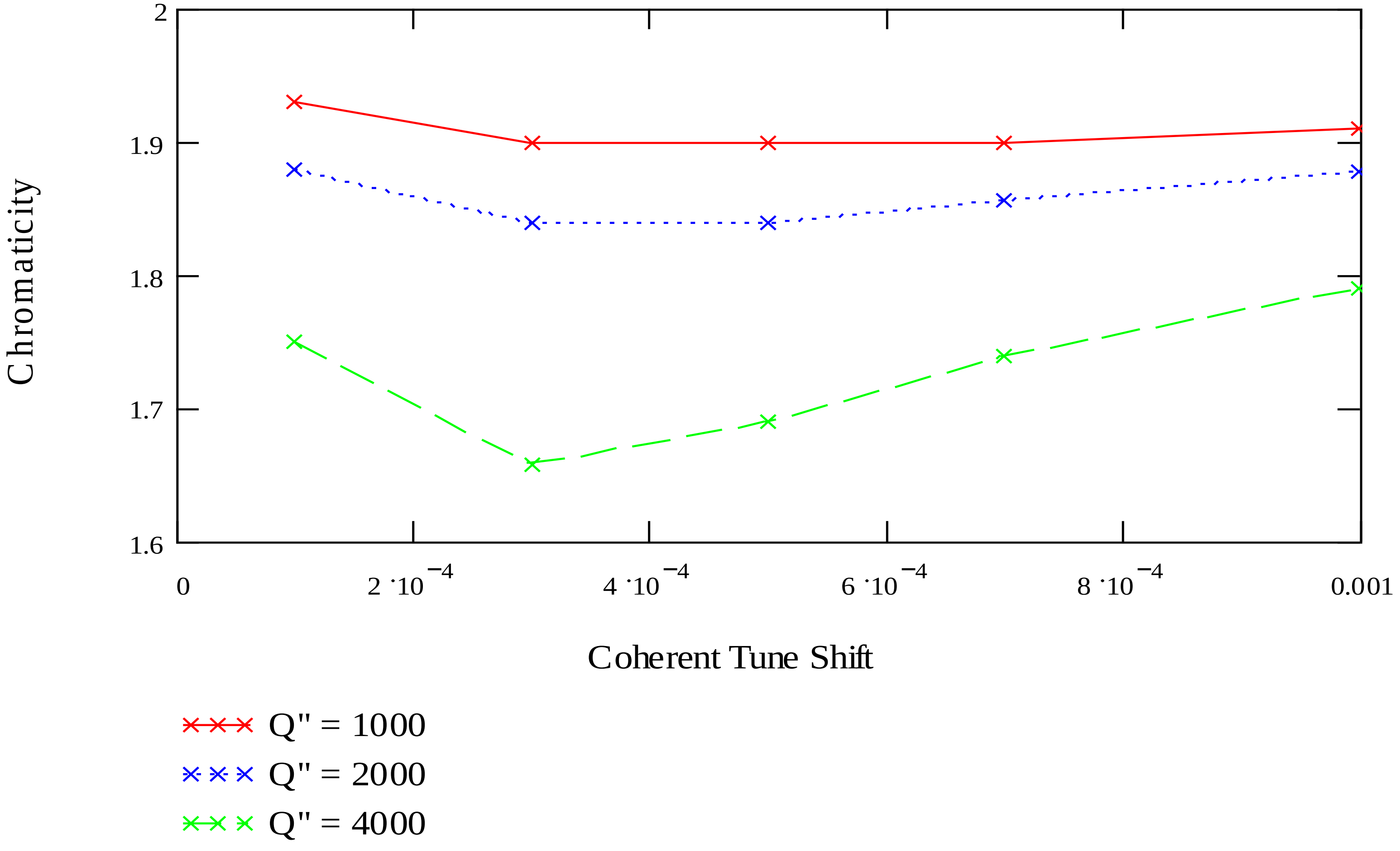
Gamma=100, sigma dp/p = 0.3e-3



Gamma=25.9, sigma dp/p = 0.3e-3



Gamma=100, sigma deltap/p=0.15e-3)



Conclusion

Based on the approximation using the Dispersion Integral solution we estimated the effect for the RHIC machine . The effect depends strongly on the strength of the 2nd order Chromaticity . Based on the observed coherent tune shift in the RHIC~ 1-3e-4 at 4000 units of 2nd Order Chromaticity the effect could be > 1 unit of Chromaticity. At more modest levels we found at 200 units of 2nd order Chromaticity the effect to be < 0.07 units. This is perhaps well within any usual measurement error. Perhaps later I can perform more through 6D simulations for the RHIC machine, but this will take considerable more effort and time to produce.

Rules of Tumb:

1. higher sigma dp/p higher effect
2. higher 2nd Order Chromaticity higher effect
3. Effect of Coherent tune shift peaks between 1 – 3 e-4 depending on Energy.